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**Brief introduction to
functional programming in Scala**

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What is a function?

We programmers often use the terms **function** and **procedure** as synonyms.

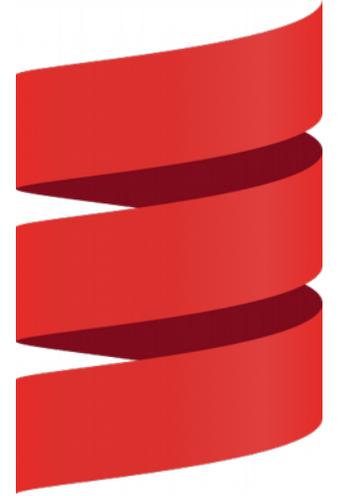
Procedure → **Subroutine** is a sequence of program instructions that perform a specific task, packaged as a unit.

Function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.



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What is Scala?

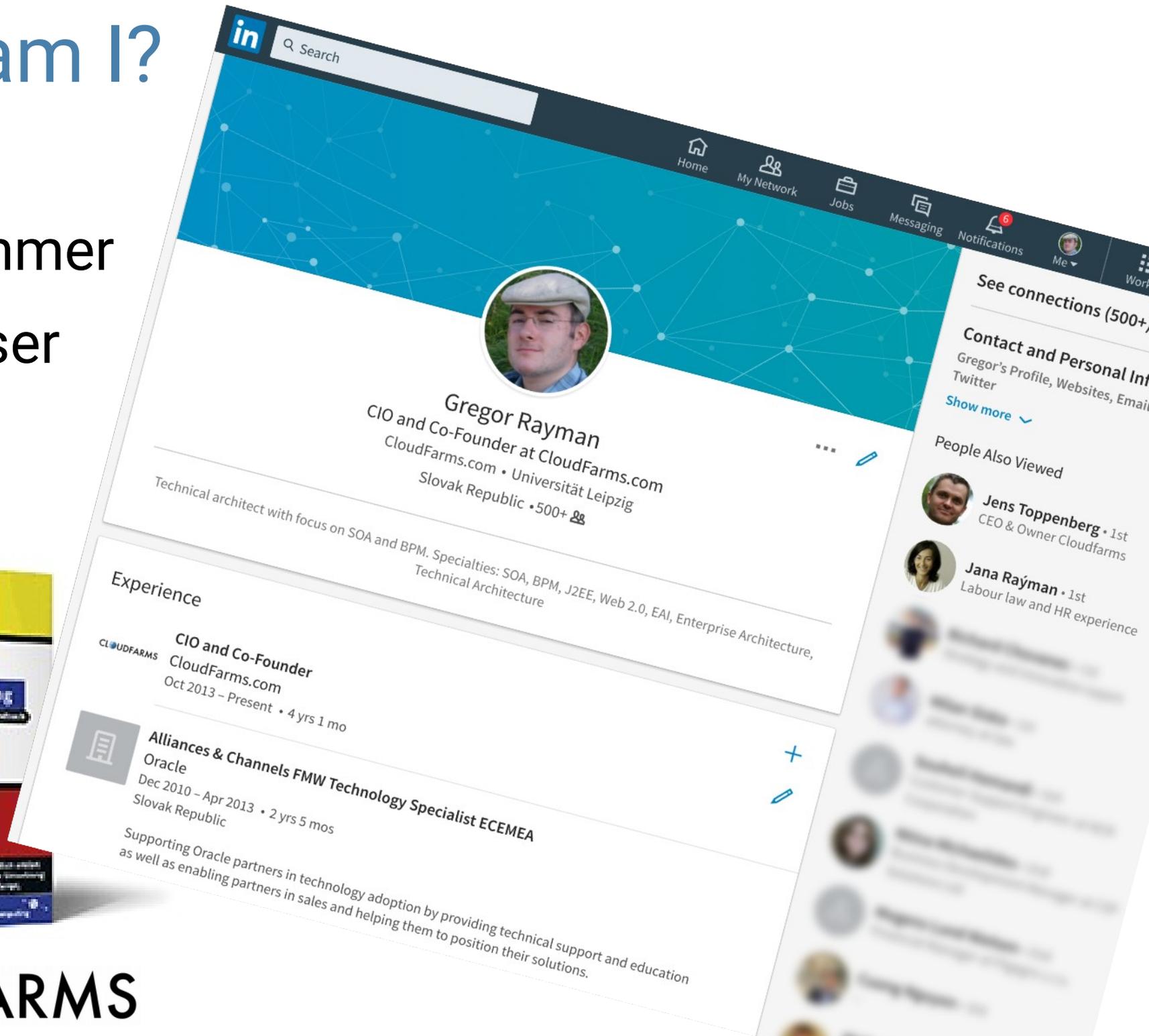


Scala

- is an object-oriented programming language
- is a functional programming language
- is a statically and strongly typed programming language
- is a scalable language
- compiles to JVM, JavaScript and native code*
- Has a lot of syntactic sugar

Who am I?

Programmer
Scala user



A bit of Scala syntax - variables

keyword	name	type (optional)	=	initial value
var	a	Int	=	42

immutable **val** b = 24

a = a + b



b = a - b

a = a - b

println(a, b)

a = "hello"



Wrong type!

A bit of Scala syntax - functions

keyword	name	parameter list(s) (optional)	return type (optional)
def	max	(a: Int, b: Int)	: Int
if	(a > b)		
	a	function's body / expression	
else			
	b		

- Note that the **if**-statement is an expression that returns a value. That is why we don't need a **return** statement. (Scala has it, don't use it)
- If the function consist of only one expression, we don't need the parentheses

A bit of Scala syntax - functions

```
def gcd(a: Int, b: Int) = {  
  var x = b  
  var y = a  
  while (x ≠ 0) {  
    val rest = y % x  
    y = x  
    x = rest  
  }  
  y.abs  
}
```

- **while** is not an expression
- A block of multiple expressions enclosed in curly braces is itself an expression. The resulting value is the value of the last one. (here the value of y)

Look, a method call on a primitive integer. In Scala everything is an object and so it has methods.

Even the operators are methods.

1+2 is the same as 1.+(2)

A bit of Scala syntax - functions

Inferred by the compiler

```
def gcd(a: Int, b: Int) : Int = {  
  var x : Int = b  
  var y = a  
  while (x ≠ 0) {  
    val rest : Int = y % x  
    y = x  
    x = rest  
  }  
  y.abs  
}
```

- **while** is not an expression
- A block of multiple expressions enclosed in curly braces is itself an expression. The resulting value is the value of the last one. (here the value of y)

A bit of Scala syntax - functions

Inferred by the compiler

```
def gcd(a: Int, b: Int) : Int = {  
  var x : Int = b  
  var y = a  
  while (x ≠ 0) {  
    val rest : Int = y % x  
    y = x  
    x = rest  
  }  
  y.abs  
}
```

```
def gcd2(a: Int, b: Int): Int =  
  if (a = 0) b.abs  
  else gcd2(b % a, a)
```

- **while** is not an expression
- A block of multiple expressions enclosed in curly braces is itself an expression. The resulting value is the value of the last one. (here the value of y)
- The result type of a recursive function has to be explicitly specified

A bit of Scala syntax - classes

Read only property

Parameter of the constructor

```
class Person(val name: String, aSurname: String) extends Mammal {  
  private var surnameNow = aSurname  
  private var spouse: Person = _  
  def surname = surnameNow  
  
  println(s"Person $name $surname was born")  
  
  def marry(p: Person, takeSurname: Boolean): Unit = {  
    if (this = p) throw new Exception("Cannot marry myself!")  
    if (spouse ≠ null) throw new Exception("Cannot marry twice!")  
    println(s"$name $surname married ${p.name} ${p.surname}")  
    if (takeSurname) surnameNow = p.surname  
    spouse = p  
    if (p.spouse ≠ this) p.marry(this, !takeSurname)  
  }  
}
```

Member variable

Public member method (note, no parameter list)

Constructor code

Another public method

**DO NOT PROGRAM THIS WAY
PLEASE!**

Substitution principle

```
val x = 7
```

```
val y: Int = 2 * x
```

```
val z: Int = x + x
```

```
var c = 0  
def x: Int = {  
  c = c + 1  
  c  
}
```

Do y and z contain the same value?

Is z odd or even?

Pure functions

A pure function:

- for the same input always returns the same value*
- the only effect it has is returning the result value. So no side effects.

```
def gcd(a: Int, b: Int) = {  
  var x = b  
  var y = a  
  while (x ≠ 0) {  
    val rest = y % x  
    y = x  
    x = rest  
  }  
  y.abs  
}
```

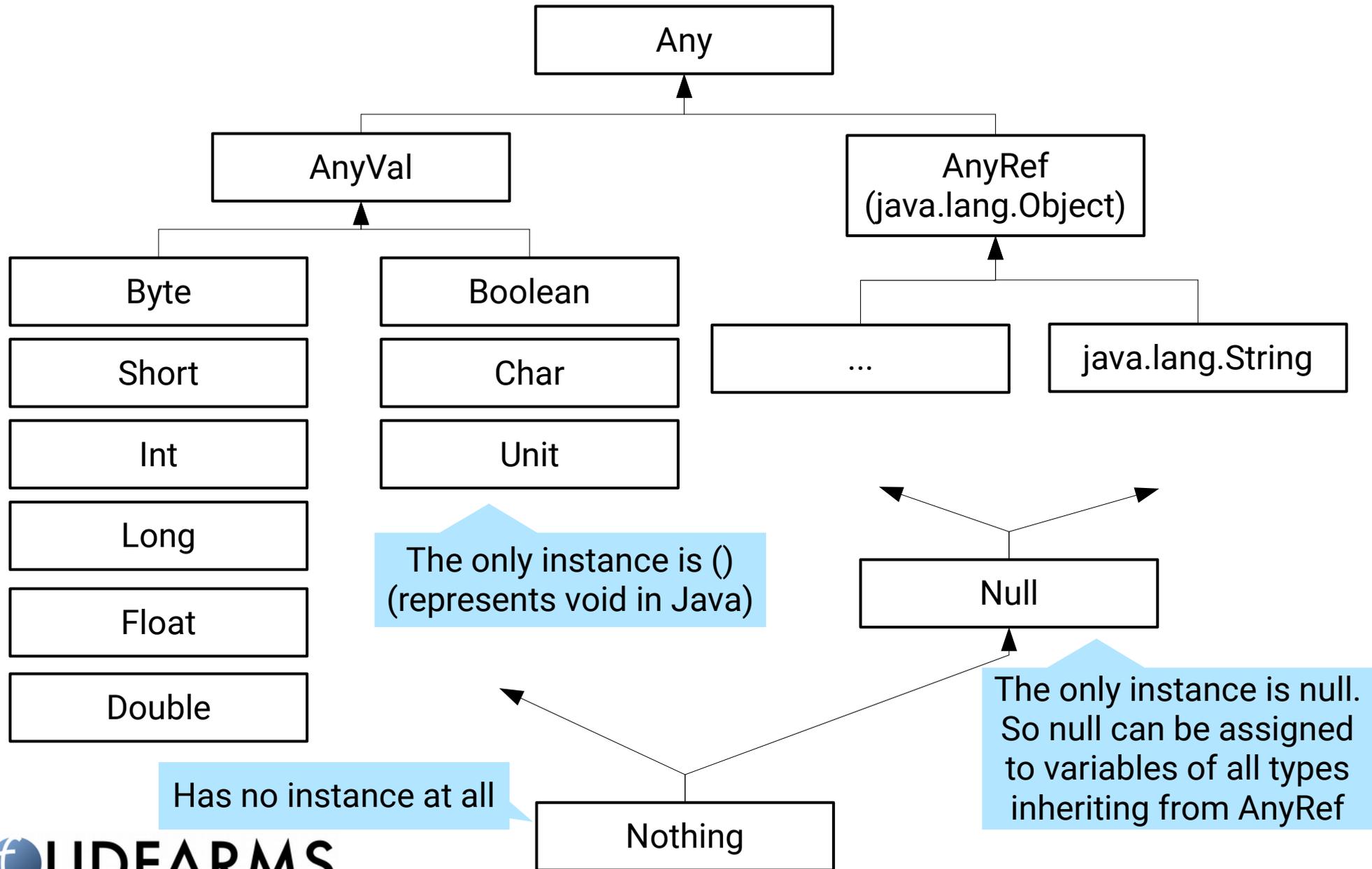
Mutable local variables are ok. This function is still pure

*) this also means that it always returns a value. So it must not throw an exception nor can it end in an endless loop

Benefits of purity and immutability

- Much simpler to reason about
- Easy to cache slowly computed functions
- Easier to use in multi-threaded environment
- Much simpler to reason about

Scala type hierarchy



Functions as first class objects

- In Scala functions are objects
- They themselves have types
- Can be assigned to variable
- Can be used as parameters of other functions
- Can be returned from functions

Anonymous function
assigned to variables

```
val up = (x: String) ⇒ x.toUpperCase  
val add = (a: Int, b: Int) ⇒ a + b  
val plus: (Int, Int) ⇒ Int = add
```

The type of add and plus is
(Int, Int) ⇒ Int

Functions have methods too

Explicitly named type of the parameter of the anonymous function

```
val withLen = (x: String) => x + x.length  
val rev: (String => String) = _.reverse
```

Explicit type of the variable `rev`

No need to give the parameter a name (used only once)

Written as a method call

```
val withLenRev = withLen.andThen(rev)  
val revWithLen = rev andThen withLen
```

Written as an operator

```
withLenRev("Scala") // returns 5alacS  
revWithLen("Scala") // returns alacS5
```

```
revWithLen.apply("Scala")
```

Functions have a method called `apply`. Scala's syntactic sugar allows you to write just the parentheses.

Functions used as parameters

```
def doTwice(f: => Unit):Unit = {f; f}
```

```
doTwice { println("Hello world") }
```

I know this is an extremely simple example.

Have you noticed the curly braces instead of parentheses?

This is a nice syntactic sugar.

You can use {x} instead of (x) if the parameter list has one parameter.

Functions used as parameters

Two parameter lists

```
def doWith(c: Closeable)(f: Closeable ⇒ Unit): Unit = {  
  try {  
    f(c)  
  } finally {  
    c.close()  
  }  
}
```

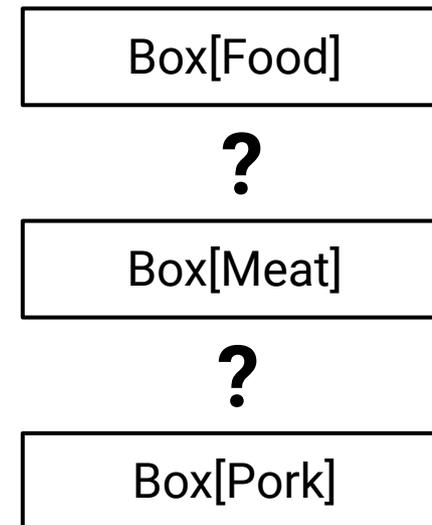
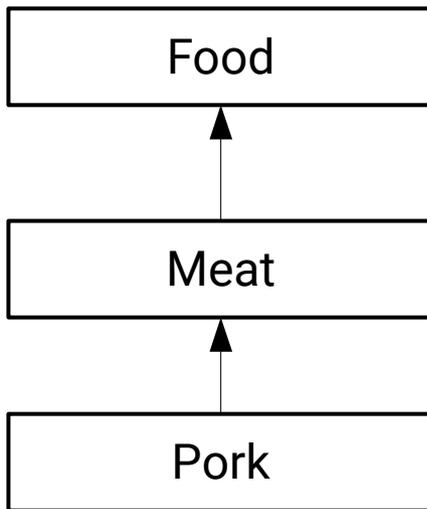
Now you can do the following. It looks like we have extended the syntax of Scala, doesn't it?

```
doWith(new FileInputStream("hello.txt")) { stream ⇒  
  ...  
}
```

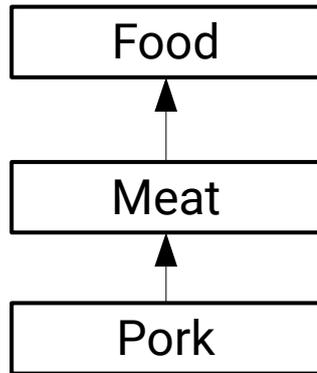
Generics and variance

Invariant type parameter

```
class Box[A] {  
  private var content: A = _  
  def put(a: A): Unit = content = a  
  def get: A = content  
}
```



Generics - Invariant



```
def examineBoxedMeat(box: Box[Meat]): Unit = {  
  val meat:Meat = box.get  
  val meat2:Meat = markInLab(meat)  
  box.put(meat2)  
}
```

Box[Pork] is ok here. Box[Food] not

Box[Food] is ok here. Box[Pork] not

```
val mealBox: Box[Food]  
val porkBox: Box[Pork]  
examineBoxedMeat( ... )
```

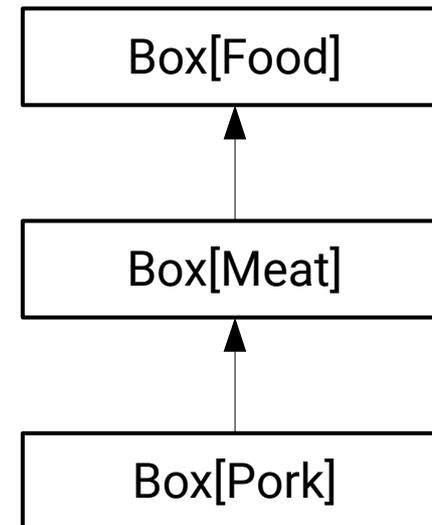
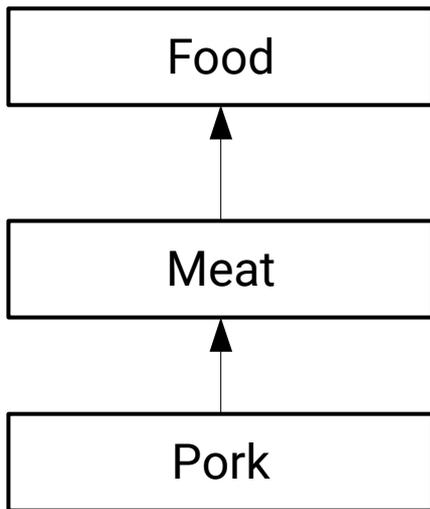
Variance - Covariant

Covariant

```
class Box[+A](content: A) {  
  def get: A = content  
}
```

```
def examineBoxedMeat(box: Box[Meat]): Boolean = {  
  val meat:Meat = box.get  
  val result:Boolean = sendToLab(meat)  
  result  
}
```

Note that `Box[+A]` can only return `A`. It cannot accept it as parameters.

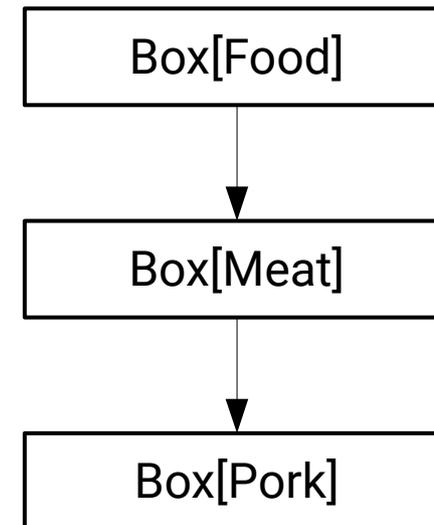
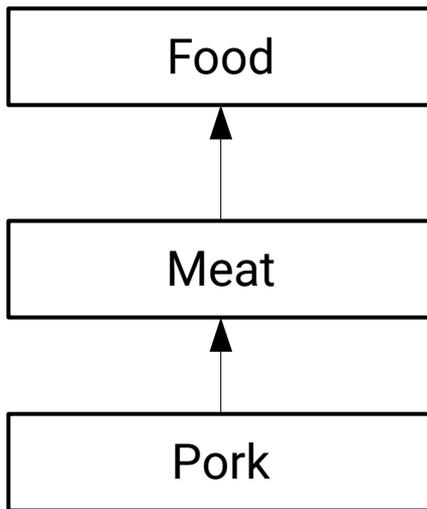


Variance - Contravariant

Contravariant

```
class Bin[-A] {  
  def getdispose(a: A):Unit = ???  
}  
  
def cleanTheFridge(bin: Bin[Meat]): Unit = {  
  val rottenMeat:Meat = getOldMeat()  
  bin.dispose(rottenMeat)  
}
```

Note that Bin[-A] methods cannot return A. It can take A as parameters.



Collections

Covariant collections can only be immutable. Let's define our own:

```
trait Collection[+A] {  
  def isEmpty: Boolean  
  def first: A  
  def rest: Collection[A]  
}
```

A **trait** is similar to Java's **interface**, it is abstract, defines capability of its instances. A **class** can implement (can inherit from) multiple **traits**.

However, **traits** can implement methods and they can also have member variables.

The simplest collections ever

A singleton instance

Is an empty collection of every type.
There is no Person in, no Bear in, no Integer in.

```
object Empty extends Collection[Nothing] {  
  override def isEmpty = true  
  override def first = throw new NotImplementedError  
  override def rest = ???  
}
```

This is a real Scala function

```
class One[+A](a:A) extends Collection[A] {  
  override def isEmpty = false  
  override def first = a  
  override def rest = Empty  
}
```

Is this useful?

“I call it my billion-dollar mistake. It was the invention of the null reference in 1965. At that time, I was designing the first comprehensive type system for references in an object oriented language (ALGOL W). My goal was to ensure that all use of references should be absolutely safe, with checking performed automatically by the compiler. But I couldn't resist the temptation to put in a null reference, simply because it was so easy to implement. This has led to innumerable errors, vulnerabilities, and system crashes, which have probably caused a billion dollars of pain and damage in the last forty years.”

Sir Charles Antony Richard Hoare

Option

All directly inheriting implementations must be in this same file

```
sealed abstract class Option[+A] {  
  val isEmpty: Boolean  
  def get: A  
}  
object None extends Option[Nothing] {  
  override val isEmpty = true  
  override def get = ???  
}
```

Look, an abstract val

What is this?

```
case class Some[+A](it: A) extends Option[A] {  
  override val isEmpty = false  
  override val get = it  
}
```

Look, a val overrides a def

The type `Option` allows us to explicitly on the type level to define, whether a variable, a parameter or a return value can be without a value.

Case classes – a bit of Scala sugar

A case class in Scala is a normal Scala class with a lot of helpful functionality automatically provided by the compiler

- All constructor parameters become read-only properties
- Automatic `toString` and `equals` implementations
- Generated `copy` methods
- Generated companion object* with useful methods

```
case class Point(x: Int, y: Int) {  
    def moveByX(dx: Int)  
        = copy(x = x + dx)  
}
```

```
val p0 = Point(0,2)  
val p10 = p0.moveByX(10)  
p10.toString // returns "Point(10, 2)"
```

*Scala does not know static methods. But it knows singleton objects. An object with the same name as a class is called a companion object. For case classes a companion object is automatically generated. It contains an factory method for creating instances of the case class:

```
object Point {  
    def apply(x: Int, y: Int)  
        = new Point(x, y)  
    ...  
}
```

Switch and If on steroids. Pattern matching

```
def describe(it: Any): String = {  
  it match {  
    case 0 ⇒ "zero"  
    case 1 ⇒ "one"  
    case x: String ⇒ x  
    case Point(0, 0) ⇒ "origin"  
    case Point(0, y) if y > 0 ⇒ s"$y up on the x axis"  
    case Point(x, y) ⇒ s"[$x,$y]"  
    case _ ⇒ it.toString  
  }  
}
```

These patterns match only a single value

Type based pattern

Type based patterns, checking properties of the case class and binding them the local variables.

The "catch all" pattern.
If no pattern matches, a run-time exception is thrown.
The compiler can actually check, whether the patterns are exhaustive. Sealed classes are needed for this functionality.

Options instead of null

```
def organizeLecture(  
    room: Room,  
    projector: Projector,  
    speaker: Person,  
    interpreter: Person  
): Lecture
```

Do we need an interpreter? Do we need a projector? Does the method signature tell us? Will we be able to organize the lecture?

```
val lecture = organizeLecture(  
    Room("C"), null, Person("Gregor"), null)
```

```
lecture.sendInvitations()
```

Really? No projector needed?

Will the function never return null?

Options instead of null

```
def organizeLecture(  
  room: Room,  
  maybeProjector: Option[Projector],  
  speaker: Person,  
  maybeInterpreter: Option[Person]  
): Option[Lecture]
```

This is clearly an allowed value

```
organizeLecture(Room("C"), None, Person("Gregor"), None)  
match {  
  case Some(lecture) ⇒ lecture.sendInvitations()  
  case None ⇒ // do nothing  
}
```

No need for a default case,
it can only be Some or None

This will be called only when the function
returns Some[Lecture]

More than one element - Lists

```
class Cons[+A](  
  override val first: A,  
  override val rest: Collection[A]  
) extends Collection[A] {  
  override val isEmpty = false  
}
```

Now we can have collections with as many elements as we want. Here a list of 3 elements:

```
val list123 = Cons(1, Cons(2, Cons(3, Empty)))
```

Note the list is constructed from the end. We start with the Empty collection and then add the elements to the head of the list.

Luckily, Scala has an implementation with more functionality and a much nicer syntax.

In Scala's collection library our first is called head, rest is called tail and Empty is Nil.

Scala Lists

You have already seen that in Scala a method with one parameter can be written as an operator. So

`1 + 2` is the same as `1.+(2)` and `f andThen g` is the same as `f.andThen(g)`

However, when the operator ends with a semicolon, it is bound to the right operand. So

`a +: b` is the same as `b.+: (a)` and `a :: b` is the same as `b::(a)`

These four lists are equal

```
List(1, 2, 3)
1 :: 2 :: 3 :: Nil
Nil.::(3).::(2).::(1)
::(3, ::(2, ::(1, Nil)))
```

Can you guess, what `::` means here?

Working with lists – Summing up

Lists can also be used in pattern matching:

```
def sum(xs: List[Int]): Int = xs match {  
  case Nil => 0  
  case h :: t => h + sum(t)  
}
```

 Danger. Stack overflow possible

@tailrec

```
def sumWithAcc(acc: Int, xs: List[Int]): Int = xs match  
{  
  case Nil => acc  
  case h :: t => sumWithAcc(acc + h, t)  
}
```

All these variants
loop over the list

```
def sumWithLoop(xs: List[Int]): Int = {  
  var acc = 0  
  var rest = xs  
  while (rest.nonEmpty) {  
    acc += rest.head  
    rest = rest.tail  
  }  
  acc  
}
```

Working with Lists - Transformation

```
def map[A,B](as: List[A])(f: A ⇒ B): List[B] =  
as match {  
  case Nil ⇒ Nil  
  case h :: t ⇒ f(h) :: map(t)(f)  
}
```

We know and use the internal structure of the list to “loop” over its elements

What will the following code return?

Look, no loop visible here

```
map(List("one", "two", "three")) { _.length }
```

Exercise: Implement a function that filters a list and returns only element for which another functions returns true. What will the type of the function be?

Note: Scala's Lists have the methods map, filter etc...

Useful methods on collections

```
trait C[A] {  
  def map[B](f: A ⇒ B): C[B]  
  def flatMap[B](f: A ⇒ C[B]): C[B]  
  def filter(p: A ⇒ Boolean): C[A]  
  def exists(p: A ⇒ Boolean): Boolean  
  def forall(p: A ⇒ Boolean): Boolean  
  def foreach(p: A ⇒ Unit): Unit  
  def find(p: A ⇒ Boolean): Option[A]  
  def reduce(op: (A, A) ⇒ A): A  
  def fold(z: A)(op: (A, A) ⇒ A): A  
  def foldLeft[B](z: B)(op: (B, A) ⇒ B): B  
  def foldRight[B](z: B)(op: (A, B) ⇒ B): B  
  def collect[B](pf: PartialFunction[A, B]): C[B]  
  ...  
  def sum: A = reduce( _ + _ )  
  def min: A = reduce( (x:A, y:A) ⇒ if (x < y) x else y )  
}
```

Can we do that?
Where do the +, < come from?

Can you guess what these methods do, just by looking at their types?

Note: This is just a simplification

map and flatMap

Task: Split a sentence (list of strings) to a list of characters codes

```
List("Hello", "World")  $\rightsquigarrow$  List(72, 101, 108, 108, 111, 87, 111, 114, 108, 100)
```

```
def strToCharCodes(s: String) = s.toList.map(_.toInt)  
def split1(ws: List[String]) = ws map strToCharCodes
```

List of lists

```
split1(List("Hello", "World"))  $\rightsquigarrow$  List(List(72, 101, 108, 108, 111), List(87, 111, 114, 108, 100))
```

```
split1(List("Hello", "World")).flatten  $\rightsquigarrow$  List(72, 101, 108, 108, 111, 87, 111, 114, 108, 100)
```

Unwraps the list of lists

```
List("Hello", "World").map( w  $\Rightarrow$  w.map(c  $\Rightarrow$  c.toInt)).flatten  
List("Hello", "World").flatMap(w  $\Rightarrow$  w.map(c  $\Rightarrow$  c.toInt))
```

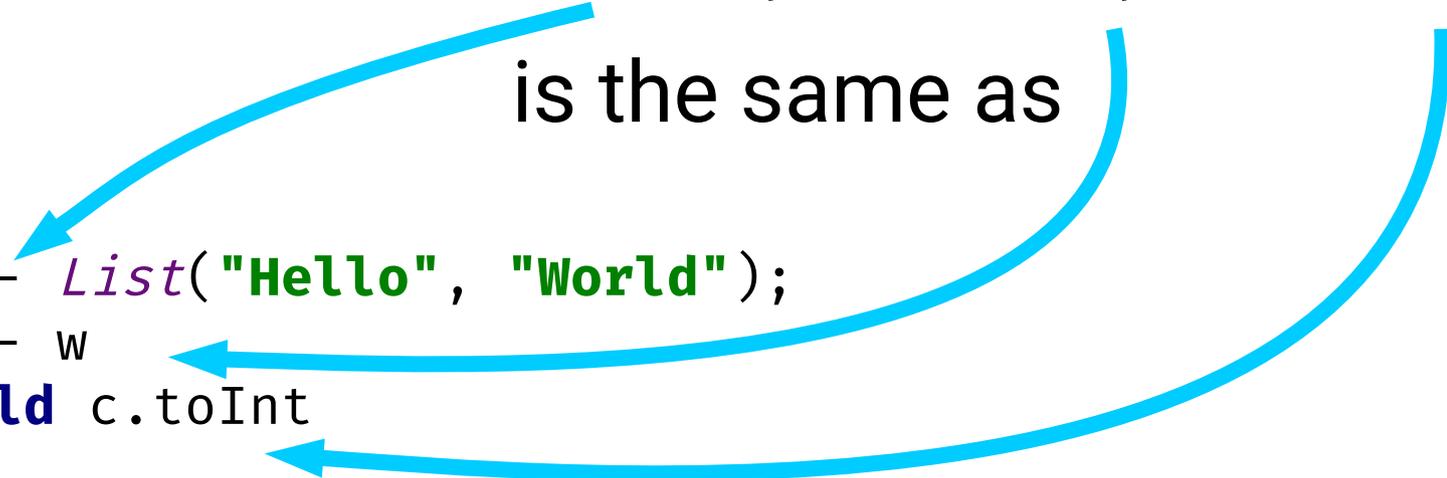
For comprehensions

In Scala there are no **for**-cycles. **for** is just syntax sugar for `map`, `flatMap`, `withFilter` and `foreach`

```
List("Hello", "World").flatMap(w => w.map(c => c.toInt))
```

is the same as

```
for (  
  w ← List("Hello", "World");  
  c ← w  
) yield c.toInt
```



For comprehensions

- Last ← before **yield** becomes map
- Last ← without **yield** becomes foreach
- Other ← become flatMap
- **if** becomes withFilter

This is not limited to collections. Any class that implements (some of) the methods, can be used in a for-comprehension.

for is just syntactic sugar

Options instead of null, again

```
def organizeLecture(  
  room: Room,  
  maybeProjector: Option[Projector],  
  speaker: Person,  
  maybeInterpreter: Option[Person]  
): Option[Lecture]  
  
for (lecture ← organizeLecture(  
  Room("C"), None, Person("Gregor"), None)  
) {  
  lecture.sendInvitations()  
}
```

This will be called only when the function returns `Some[Lecture]`. No need to write an empty clause when it returns `None`.

Useful methods (on collections?)

```
trait C[A] {  
  def map[B](f: A ⇒ B): C[B]  
  def flatMap[B](f: A ⇒ C[B]): C[B]  
  
  ...  
}
```

C stands for Context. The function `f` called by `map` does not need to know anything about the structure of `C`. The function used in `flatMap` knows about `C`, so that we cannot combine incompatible contexts.

```
for (  
  person ← personContext;  
  meal ← mealContext if !meal.meat || !person.vegetarian  
) yield (person.name, meal.name)
```

This works if both `personContext` and `mealContext` are of the same kind (both `Collection`, or both `Future`, etc).

Using **for** for some random stuff :)

```
trait Rnd[+A] {  
  def next(): A  
}
```

```
object RndDouble extends Rnd[Double] {  
  override def next(): Double = Math.random()  
}
```

```
class RndInt(from: Int, to: Int) extends Rnd[Int] {  
  override def next(): Int =  
    from + ((to - from + 1) * RndDouble.next).floor.toInt  
}
```

```
val tenGenerator = new RndInt(1, 10)
```

Does this function really
need to know that it
deals with random numbers?

Using **for** for some random stuff :)

```
trait Rnd[+A] { self ⇒  
  def next(): A  
  final def map[B](f: A ⇒ B) = new Rnd[B] {  
    override def next(): B = f(self.next())  
  }  
}
```

`self` is just an alias for **this**, because **this** in the `Rnd[B]` references the anonymous nested class

```
val tenGenerator = RndDouble map { n ⇒  
  1 + (10 * n).floor.toInt  
}
```

This function does not know that `n` is a random number

Using **for** for some random stuff :)

```
trait Rnd[+A] { self =>
  def next(): A
  final def map[B](f: A => B) = new Rnd[B] {
    override def next(): B = f(self.next())
  }
  final def flatMap[B](f: A => Rnd[B]) = new Rnd[B] {
    override def next(): B = f(self.next()).next()
  }
}
```

```
val moveGenerator = for (
  c ← tenGenerator;
  col = ('A' + c).toChar;
  row ← tenGenerator
) yield (col, row)
```

flatMap combines this Rnd
with the one returned from f

```
moveGenerator.next() // returns (F,2) or (B,4) or ...
```

Using **for** for some random stuff :)

We have created a generator of (Char, Int) pairs.
Can we also create a generator of Int sequences?

```
val seqGenerator = for (  
  i ← 1 to 10;  
  n ← tenGenerator  
) yield n
```



Does not work! We cannot
combine a Range with a Rnd
this way

```
class IntSeqRnd(len: Int) extends Rnd[Seq[Int]] {  
  override def next() =  
    for (i ← 1 to len) yield tenGenerator.next()  
}
```

But this is not functional!

A pure function:

- for the same input always returns the same value
- the only effect it has is returning the result value.
So no side effects.

`Math.random()` certainly is not a pure function. It does not always return the same result value (that would make it be quite pointless) and calling it changes some internal **state** of the pseudo-random generator, so it has side effects.

Can we have a pure function that can provide random numbers?

Modeling state functionally

- In object oriented programming state is modeled as objects. State changes are modeled as changing the data of object's member variables.
- In (pure) functional programming, data is immutable.

```
class Person(n:String) {  
  private var name = n  
  def getName = name  
  def setName(nn:String):Unit = {name = nn}  
}
```

```
val p = new Person("Gregor")  
p.setName("Greg")  
p.getName
```

The state of the object p
has been changed

```
case class Person(name: String)
```

```
val p = Person("Gregor")  
val p1 = p.copy(name = "Greg")
```

The object p is not changed.
New state is in the new object p1

“Pure?” functional random numbers

```
class FunRandom extends Function0[(Double, FunRandom)] {  
  private val n = Math.random()  
  private lazy val next = new FunRandom  
  def apply(): (Double, FunRandom) = (n, next)  
}
```

```
val f0 = new FunRandom  
val (n1a, f1a) = f0()  
val (n1b, f1b) = f0() // n1a == n1b  
val (n2a, f2a) = f1a()  
val (n2b, f2b) = f1b() // n2a == n2b
```

FunRandom always returns the same result. It returns a pair of a random number and another instance of RunRandom.

It is still not pure, because creating the new instance has side effects on the state of Math.random.

Is there a way? Can a real world program be functionally pure?

Useful programs have to interact with the outside world. They have to have have inputs, outputs. So totally pure programs are not really useful. But we can “push” the impure , state changing functionality to the borders of the programs. (To learn more about this, study the IO monad)

Note: Yes, it is possible to create a purely functional pseudo-random generator by keeping the state inside the function instances.

Type classes

Remember?

```
trait C[A] {  
  ...  
  def sum: A = reduce( _ + _ )  
  def min: A = reduce( (x:A, y:A) => if (x < y) x else y )  
}
```

Can we do that?
Where do the +, < come from?

Type classes

Let's write a function that finds the smallest element. To be able to do that, we need a decision function that tells which from two elements is smaller.

```
trait LessThan[-T] {  
  def lt(a:T, b:T): Boolean  
}
```

A Person does not have any "natural" ordering

```
case class Person(name: String, age: Int, height: Int)
```

```
val ageLessThan: LessThan[Person] = new LessThan[Person] {  
  override def lt(a: Person, b: Person) = a.age < b.age  
}
```

Using this, we will sort people by age

Type classes

```
val ageLessThan: LessThan[Person] = new LessThan[Person] {  
  override def lt(a: Person, b: Person) = a.age < b.age  
}
```

```
def least[T](a: T, b: T)(lessThan: LessThan[T]) =  
  if (lessThan.lt(a, b)) a else b
```

```
Least(  
  Person("Gregor", 47, 189),  
  Person("Vincent", 7, 130)  
) (ageLessThan)
```

The function `least` will be applicable to any type `T` for which we can provide an instance of `LessThan[T]`

Scala has a very powerful feature called implicit parameters. It instructs the compiler to automatically use implicit variables whenever we have not specified one explicitly.

Let's use this feature to simplify our code

Type classes

```
implicit val ageLessThan: LessThan[Person] =  
  new LessThan[Person] {  
    override def lt(a: Person, b: Person) = a.age < b.age  
  }
```

```
def least[T](a: T, b: T)(implicit lessThan: LessThan[T]) =  
  if (lessThan.lt(a, b)) a else b
```

```
Least(  
  Person("Gregor", 47, 189),  
  Person("Vincent", 7, 130)  
)
```

No need to provide the parameter list explicitly

Note that this works only, when the implicit parameter can be selected unambiguously.

There is even more concise way to write the function `least[T]`

Type classes

This notation means the same as the previous one.
The function automatically gets another parameter list with an anonymous parameter of the type `LessThan[T]`
We say that `T` belongs to the type class `LessThan`.

```
def least[T:LessThan](a: T, b: T) = {  
  val lessThan = implicitly[LessThan[T]]  
  if (lessThan.lt(a, b)) a else b  
}
```

To access the parameter by name,
we use the helper method *implicitly*

Let's find the smallest element from more than two elements.

Type classes, Higher kinded types

This means, that `C` needs a parameter. `C[T]` is a type, `C` is called a type constructor.

```
trait Reducer[-C[_]] {
  def reduce[T](c: C[T])(f: (T, T) => T): T
}

implicit val seqReducer: Reducer[TraversableOnce] = new Reducer[TraversableOnce] {
  override def reduce[T](c: TraversableOnce[T])(f: (T, T) => T) = c.reduce(f)
}

def min[C[_]:Reducer, T:LessThan](c:C[T]) = {
  val reducer = implicitly[Reducer[C]]
  val lessThan = implicitly[LessThan[T]]
  reducer.reduce(c)(least[T])
}

min(List(
  Person("Gregor", 47, 189),
  Person("Vincent", 7, 130),
  Person("Adam", 4, 101)))
```

